Innovations in the design of bundled-item auctions

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Auctions have captured the attention of economists for several reasons. One is the relevance for specific applications of research on auctions in light of their widespread and growing use in the field, particularly in online transactions. Auctions are also highly amenable to economic analysis, because the rules are generally precisely defined; modern game-theoretic techniques of economic analysis can be brought to bear with fewer simplifying assumptions than for many other areas of economics. Furthermore, the rich variety of possible auction rules and underlying environments in which auctions can be conducted provide a wealth of interesting economic issues to investigate. The study of auctions, which began with the seminal work of Vickrey (1), has yielded useful policy prescriptions with regard to the sale of goods. The work that Porter *et al.* (2) report and summarize in this issue of PNAS is representative of a particularly interesting and useful branch of this research, the development of combinatorial auctions.

The Porter *et al.* (2) study employs the research methodology of experimental economics. Although introduced later than in the natural sciences and psychology, experimental methods in economics have gained broad acceptance in recent decades. As in the natural sciences, an experiment involves constructing a laboratory environment specifically for the purpose of addressing research questions. Human subjects are placed in a laboratory economy, and their decisions and the resulting outcomes are studied. The investigator is able to observe variables with values that are unknown in typical economic situations, to control parameters of interest, and to replicate the experiment repeatedly under identical conditions. One of the most innovative uses of the methodology has been to design and test new auction rules for use in specific applications.

The performance of an auction system is typically evaluated on two criteria: efficiency and revenue. Economists generally view an allocation of an item to the potential buyer who receives the highest value from obtaining it, and is therefore willing to pay the most for it, as a desirable outcome. This allocation is termed the efficient outcome. The other criterion, particularly important to sellers, is the revenue the auction generates. Experiments allow precise measurement of these variables and thus enable comparisons of the performance of different systems under otherwise identical conditions.

Although the primary research focus has been on auctions of a single item, many existing auctions involve the sale of multiple items. Considerable progress in theoretical modeling has been made for cases where all the goods sold are identical and the value of obtaining extra units does not increase as one obtains more (3–5). Auctions for government debt, produce, hotel rooms, or airplane tickets often have this property. Experimental work has explored the properties of various auction types for this case (see, for example, refs. 6–9).

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However, the existence of a synergy in preferences between items presents challenges for theoretical modeling. Consider, for example, the value of obtaining a particular day off from work. A Thursday off may be more valuable for someone who also has Friday off, because it allows a wider range of weekend activities. The value for one good increases if a complementary good is obtained. Some goods that are or might be sold by auction have this property. Ausubel and Cramton (10) and Bykowsky *et al.* (11) argue that synergies existed in the multibillion-dollar broadband personal communications service auctions that the Federal Communications Commission conducted in the mid-1990s. The value to bidders for a broadcast license covering one region depended on whether they could obtain licenses in adjacent regions. Experimental economists have also explored the operation of markets in the context of other goods with synergies, such as takeoff and landing slots at different airports (12, 13), scheduling on different segments of a network of train tracks (14), and allocation of resources on the space station (15).

Auctioning each item separately to the highest bidder presents a hazard to bidders called the exposure problem (16, 17), which arises when a bidder is able to obtain only part of but not the entire "package" of items that has value to him/her. For example, suppose that for a particular bidder, called i, the value of good A is zero. The value for good B is also zero. However, the value for obtaining both together is \$20,000. If i is outbid for one of the units or refuses to bid out of fear of failing to also win the second unit, inefficiency can result if the efficient outcome would assign both items to i.

To assign items under such conditions efficiently, combinatorial auctions that permit "package" bidding have been developed. For example, i can bid on a package of units consisting of both A and B and attach a condition that if both cannot be obtained, he/she obtains neither and pays nothing. The space of possible bids is augmented to accord better with participants' preferences and facilitate attaining an efficient outcome. For example, in the auction studied in ref. 12, all bidders submit bids for packages simultaneously. An integer program, a linear program in which elements of the solution are constrained to take on integer values, is solved. The algorithm allocates packages to bidders in a manner that maximizes the sum of the bids, subject to the constraints bidders specify on the composition of packages as well as constraints on feasibility (that an item cannot be divided and can be allocated to only one bidder). The prices charged for each item are those in the solution to a pseudodual of the integer program.

However, in a combinatorial auction, two types of complexity arise that have potential to hinder system operation. The first is computational complexity (18). The algorithms that calculate outcomes and prices are typically integer programs that are nondeterministic polynomial complete and become intractable as the scale of the problem, the number of bidders and potential packages, increases. The second type is cognitive complexity. The large space of possible bids (there are $2^n - 1$ possible packages for n items) means that the cognitive demands on participants can be overwhelming. Multiround, iterative

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combinatorial auctions that provide bidders with feedback about current prices do seem to help bidders to formulate better strategies (19) but may be even more computationally intensive because they require solution of an integer program in each round.

The adaptive user selection mechanism (AUSM), introduced by Banks *et al.* (15), offers one solution to the computational complexity problem. The AUSM is a continuous process in which any participant may submit a bid for any package at any time. A bid is provisionally accepted and becomes the standing bid if it is greater than all current standing bids for overlapping packages. For example, if the current standing bid for a package consisting of A and B is $500 and that for the package consisting of C and D is $700, a prospective bidder *i* must bid $1,201 to obtain the package consisting of B and C. The auction ends when no agent wishes to bid or when the auctioneer closes the market. This simple dynamic process requires no complicated programming problems to be solved.

However, the AUSM process is vulnerable to the threshold problem (see ref. 16). Efficiency may dictate that multiple bidders must coordinate their activity. Consider the following example. Suppose that item A is worth $1,000 to bidder *i* and item B is worth $800 to bidder *j*. Obtaining both A and B is worth $1,200 to bidder *k*, but obtaining only one of the two items has no value. If *k* submits a bid of $1,001 for the package consisting of both A and B, then neither *i* nor *j* can overbid *k* without committing to obtaining an item at a price greater than its value to them. The threshold problem can be overcome in principle by allowing *i* and *j* to combine their bids and displacing *k* with the joint bid. However, this induces an element of conflict between *i* and *j*, who each would like to lower the share they contribute to the joint bid. For example, *i* would like to overbid the $1,001 of *k* by contributing $203 and having *j* contribute $799, whereas *j* would like to contribute a smaller share. The joint-value factor (2) is a measure of the potential difficulty of the threshold problem. The existence of the own effect, where bidder *k*'s bid for a larger package can only be displaced if *k* combines with at least one other bidder, exacerbates the threshold problem.

The combinatorial clock auction is designed to overcome the potential pitfalls discussed here. An ascending price "clock" mechanism (20) establishes prices. Bidders do not submit prices but respond to prices the mechanism announces. This is viewed as desirable because the ability of bidders to submit their own prices is believed to facilitate collusion in that it creates an opportunity for bidders to communicate through the amounts of their bids (21). In the combinatorial clock auction, bidders indicate whether they wish to buy particular items and packages at current prices, with a constraint that the bidder commits to purchase one item in a package only if the entire package can be purchased. If a unit is included in more than one demanded package, its price is raised by a small increment. When prices reach a level where exactly one player claims each item, the auction ends and the items are allocated to current claimants at the current prices. If an item becomes unclaimed, an integer program maximizes the sum of bids over feasible allocations, searching over all current and previous claims.

The auction is not cognitively burdensome for bidders, who merely send a series of "yes" or "no" messages. The computational burden is light in that solution of integer programs is required only in some cases. Combinatorial bidding overcomes the exposure problem. The fact that prices are posted rather than bidders submitting bids minimizes the communication possibilities between bidders. The pricing of each individual item avoids the threshold problem. The experimental results indicate a strong tendency for the process to generate efficient outcomes. The work provides an illustration of how economic experiments can be used to develop systems for use in actual practice and how such development proceeds by devising techniques to mitigate weaknesses of previous designs.

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